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Induced motion of domain walls in multiferroics

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Abstract

We study the dynamics of a 180° domain wall of **ab**-type in external alternating magnetic and electrical fields in magnetic materials with linear magnetoelectric interaction. We discuss the features of oscillatory and drift motion of domain wall and stripe structure depending on the parameters of external fields and characteristics of the material.

1. Introduction

The magnetic domains and domain walls (DW) stipulate the nonuniformity of a magnetic material and therefore affect its basic physical properties [1]. Thus, investigations of the dynamical properties of magnetic nonuniformities are of great interest now. The investigations of magnetic domain structure (DS) and DW in multiferroics in electrical fields are now of great interest also from the applied standpoint [2].

The motion of DW can be induced by a stationary [3] or variable [4, 5] magnetic field, by sound [6], by laser beam [7], etc. The influence of magnetic field on the dynamics of DW has been studied most of all. The influence of the rest of the factors (electrical field, etc) has been less frequently investigated.

The influence of a *stationary* electrical field on the density of DW surface energy and the velocity of its motion in Seignette magnetics were studied in [8]. In works [9, 10] the effect of *variable* electrical field on the dynamics of DW was investigated. In the case of the spin-reorientation first-order phase transition of Morin's type in rhombic Seignette antiferromagnetics the magnetoelectric interaction excites oscillations of 90° DW. The amplitude of oscillation velocity of such DW is proportional to the electric field amplitude [8]. The drift of a 180° DW of **ac**-type [10] occurs in magnetics with a linear magnetoelectric interaction under the influence of external electric and magnetic fields. The drift velocity in this case is proportional to the square of the amplitude of variable fields.

The controlled DW displacement under the influence of a *stationary* electrical field in garnet ferrite films was observed experimentally in [11]. The direction of DW displacement changes into the opposite one for the change of the polarity of the electric field. The nonuniform magnetoelectric effect was proposed as a mechanism for the observable phenomenon. The experimental observations of dynamical transformations of a magnetic stripe-domain structure in a bilayer thin film ferromagnetic–Ni/ferroelectric–lead zirconate titanate heterostructure in an electric field are presented in [12].

In the present paper we study analytically the nonlinear dynamics of **ab**-type DW in a magnetic with linear magnetoelectric interaction. As the subject of the investigation we used a two-sublattice system of a weak ferromagnetic (WFM) [8–10] which can describe the magnetic subsystem of rhombic Seignette magnetics [13] (for example, Ni–Cl-boracites or rare-earth manganites of crystalline class C_{2v}).

2. The model and equations of motion

Let us write the Lagrange density function $L(\mathbf{l})$ of a two-sublattice weak ferromagnet in terms of the unit antiferromagnetic vector $\mathbf{l}$, $\mathbf{l}^2 = 1$ [2, 14]:

$$L(\mathbf{l}) = M_0^2\left[\frac{\alpha}{2c^2}(\dot{\mathbf{l}})^2 - \frac{\alpha}{2}(\nabla\mathbf{l})^2 - \left(\frac{\beta_1}{2}l_z^2 + \frac{\tilde{\beta}_2}{2}l_y^2\right) + \frac{2d}{\delta}(h_x l_z - h_z l_x) - w_{\mathrm{me}}(\mathbf{l}) + \frac{4}{\delta g M_0}(\mathbf{h}\cdot[\dot{\mathbf{l}}\times\mathbf{l}]) - \frac{2}{\delta}(\mathbf{l}\cdot\mathbf{h})^2\right], \quad (1)$$

where $\dot{\mathbf{l}}$ denotes the derivative with respect to time; $M_0^2 = (M_1^2 + M_2^2)/2$; M_0 is the length of the sublattice magnetization vector; $c = gM_0\sqrt{\alpha\delta}/2$ is the minimum spin-wave phase velocity; δ and α are the homogeneous and inhomogeneous exchange coupling constants, respectively; g is the gyromagnetic ratio (equal for each sublattice); β_1 and $\tilde{\beta}_2$ are the effective constants of rhombic anisotropy; $\tilde{\beta}_2 = \beta_2 + \frac{d^2}{\delta}$; d is the exchange-relativistic Dzyaloshinskii constant (as shown in [15], a weak ferromagnetism occurs in the plane perpendicular to the pyroelectric axis C_2 which is considered to be directed along the Y axis); $w_{\mathrm{me}}(\mathbf{l})$ is the energy density of the magnetoelectric interaction; $\mathbf{h} = \boldsymbol{H}/M_0$; $\boldsymbol{H} = \boldsymbol{H}_0\cos(\omega t + \boldsymbol{\chi})$ is the external alternating magnetic field with frequency ω and phase shift $\boldsymbol{\chi}$.

Let us direct the external electric field $\boldsymbol{E}(t) = \boldsymbol{E}_0\cos(\omega t)$ along the pyroelectric axis. As E_y is transformed by the identical representation of pyroelectric class C_{2v}, we have for $w_{\mathrm{me}}(\mathbf{l})$ the same form as for the energy density of magnetic anisotropy but with other phenomenological constants:

$$w_{\mathrm{me}}(\mathbf{l}) = E_y(t)\left(\frac{b_1}{2}l_z^2 + \frac{b_2}{2}l_y^2\right), \tag{2}$$

where b_1 and b_2 are the constants of the magnetoelectric interaction.

The dynamic stopping of the DW, due to dissipative processes, will be taken into account by using the dissipative function

$$F = \frac{\lambda M_0}{2g}\dot{\mathbf{l}}^2, \tag{3}$$

where λ is the dimensionless Gilbert damping constant.

Since the components of the vector $\mathbf{l}$ are connected by the relation $\mathbf{l}^2 = 1$, it is convenient to rewrite the Lagrange density function (1) in terms of two independent angle variables θ and φ which parametrize the unit vector $\mathbf{l}$:

$$l_x + \mathrm{i}l_z = \sin\theta\exp(\mathrm{i}\varphi), \qquad l_y = \cos\theta. \tag{4}$$

Taking into account the parametrization from equation (4), we obtain from the Lagrange density function (1) the equations of motion for the angle variables θ and φ:

$$\begin{aligned}
&\alpha\left(\Delta\theta - \frac{1}{c^2}\ddot{\theta}\right) + \sin\theta\cos\theta\left[\alpha\left(\frac{1}{c^2}(\dot{\varphi})^2 - (\nabla\varphi)^2\right)\right.\\
&\quad\left. + (\tilde{\beta}_2 + b_2E_y) - (\beta_1 + b_1E_y)\sin^2\varphi\right]\\
&\quad + \frac{2d}{\delta}(h_x\sin\varphi - h_z\cos\varphi)\cos\theta\\
&\quad - \frac{4}{\delta}((h_x\cos\varphi + h_z\sin\varphi)\sin\theta + h_y\cos\theta)\\
&\quad \times (h_x\cos\theta\cos\varphi - h_y\sin\theta + h_z\cos\theta\sin\varphi)\\
&\quad + \frac{4}{\delta gM_0}[\dot{h}_x\sin\varphi - \dot{h}_z\cos\varphi + h_y\dot{\varphi}\sin 2\theta\\
&\quad + 2\dot{\varphi}\sin^2\theta(h_z\sin\varphi + h_x\cos\varphi)] = \frac{\lambda}{gM_0}\dot{\theta},
\end{aligned} \tag{5}$$

$$\begin{aligned}
&\alpha\nabla(\sin^2\theta(\nabla\varphi)) - \frac{\alpha}{c^2}\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\varphi}\sin^2\theta)\\
&\quad - (\beta_1 + b_1E_y)\sin^2\theta\sin\varphi\cos\varphi\\
&\quad - \frac{4}{\delta}[(h_x\cos\varphi + h_z\sin\varphi)\sin\theta + h_y\cos\theta]\\
&\quad \times (h_z\cos\varphi - h_x\sin\varphi)\sin\theta\\
&\quad + \frac{2d}{\delta}(h_x\cos\varphi + h_z\sin\varphi)\sin\theta\\
&\quad + \frac{4}{\delta gM_0}[(\dot{h}_x\cos\varphi + \dot{h}_z\sin\varphi)\sin\theta\cos\theta - \dot{h}_y\sin^2\theta\\
&\quad - h_y\dot{\theta}\sin 2\theta - 2\dot{\theta}\sin^2\theta(h_z\sin\varphi + h_x\cos\varphi)]\\
&= \frac{\lambda}{gM_0}\dot{\varphi}\sin^2\theta.
\end{aligned} \tag{6}$$

If $\beta_1, \tilde{\beta}_2 > 0$, then in the absence of external fields in the homogeneous ground state the vector $\mathbf{l}$ is collinear to the x axis (a axis of the crystal). In this case it can be easily seen that the equations of motion have two particular classes of nontrivial solutions describing two types of 180° DW which can exist in the magnet under consideration [2, 14]. The first class of solutions describes the case when the vector $\mathbf{l}$ rotates in the XZ plane (**ac**-type DW) and the second one describes the rotation of the vector $\mathbf{l}$ in the XY plane (**ab**-type DW).

In the case of $\beta_1 > \tilde{\beta}_2 > 0$ the DW of the **ab**-type is stable. This DW corresponds to $\varphi = \varphi_0 = 0$, and the angle variable $\theta = \theta_0(y)$ satisfies the equation

$$\alpha\theta_0'' + \tilde{\beta}_2\sin\theta_0\cos\theta_0 = 0 \tag{7}$$

and boundary conditions $\theta_0(\pm\infty) = \pm\pi/2$. We consider the distribution of magnetization to be nonuniform along the Y axis (the prime denotes differentiation with respect to this coordinate).

The solution of equation (7) that describes the static 180° DW has the following form:

$$\begin{aligned}
\theta_0' &= -\frac{R}{y_0}\cos\theta_0(y) = -\frac{R\rho}{y_0}\sec h\left(\frac{y}{y_0}\right),\\
\sin\theta_0(y) &= -R\tanh\left(\frac{y}{y_0}\right),
\end{aligned} \tag{8}$$

where $y_0 = \sqrt{\alpha/\tilde{\beta}_2}$ is the DW thickness, R is the topological DW charge and ρ is the parameter that describes the direction of rotation of the vector $\mathbf{l}$ in the DW.

As is known, the neighbouring 180° DW separating the domains with the opposite magnetization axis direction in a stripe-domain structure (SDS) possess the opposite topological charges $R = \pm 1$. These charges are determined by the boundary conditions of equation (7).

In turn, the rotation of the vector $\mathbf{l}$ in various DW can be about either a positive or a negative direction of the Y axis. This direction of rotation is determined by the parameter $\rho = \pm 1$. Therefore, the neighbouring DW of **ab**-type in the SDS correspond to

$$l_x(y \to \pm\infty) = \mp R, \qquad l_y(y = 0) = \rho.$$

The isolated static DW is described by the relations (8) with $R = -1$ and $\rho = +1$.

3. The solution of equations of motion

For the description of nonlinear macroscopic DW dynamics we use one of the perturbation theory versions for solitons [4–6].

We introduce a collective variable $Y(t)$ which has the meaning of the coordinate of the DW centre at the instant of time t, the derivative of which defines the instantaneous velocity of DW $V(t) = \dot{Y}(t)$. The DW drift velocity is defined as the instantaneous DW velocity $V(t)$ averaged over the oscillation period $V_{\rm dr} = \bar{V}(t)$ (the bar denotes averaging over the external field oscillation period). Assuming the amplitude of the external electric E_y and magnetic $\mathbf{h}$ fields to be small, we represent the functions $\theta(y,t)$, $\varphi(y,t)$ and $V(t)$ by series in powers of the field amplitude:

$$\begin{aligned}\theta(y,t) &= \theta_0(\xi) + \theta_1(\xi,t) + \theta_2(\xi,t) + \cdots,\\ \varphi(y,t) &= \varphi_1(\xi,t) + \varphi_2(\xi,t) + \cdots,\\ V &= V_1(t) + V_2(t) + \cdots,\end{aligned} \tag{9}$$

where $\xi = y - Y(t)$; subscripts $n = 1, 2, \ldots$ denote the order of smallness of the quantity to the field amplitude θ_n, φ_n, $V_n \sim h^n$, E_y^n. The function $\theta_0(\xi)$ describes the motion of an undistorted DW (the structure of $\theta_0(\xi)$ is the same as that of $\theta_0(y)$ in the static solution (8)). The functions of higher orders $\theta_n(\xi,t)$ and $\varphi_n(\xi,t)$, $n = 1, 2, \ldots$ describe the distortions of the DW shape and the excitation of spin waves.

We substitute the expansions (9) in equations (5) and (6) and separate terms of different orders of smallness. Obviously, in the zero approximation we obtain equation (7) which describes a DW of **ab**-type at rest.

The perturbation theory first-order equations can be written in the form

$$\begin{aligned}(\hat{L} + \hat{T})\theta_1(\xi,t) &= \frac{b_2}{\tilde{\beta}_2} E_y \sin\theta_0 \cos\theta_0 - \frac{4}{\tilde{\beta}_2 \delta g M_0}\dot{h}_z\\ &- \frac{\cos\theta_0(\xi)}{y_0\omega_1^2}\left(R(\dot{V}_1 + \omega_r V_1) + \frac{(gM_0)^2}{2} y_0 h_z d\right),\end{aligned} \tag{10}$$

$$\begin{aligned}(\hat{L} + \sigma + \hat{T})\mu_1(\xi,t) &= \frac{2d}{\tilde{\beta}_2\delta} h_x\\ &+ \frac{4}{\tilde{\beta}_2 \delta g M_0}[\dot{h}_x \cos\theta_0(\xi) - \dot{h}_y \sin\theta_0(\xi)],\end{aligned} \tag{11}$$

where we denote

$$\mu_1(\xi,t) = \varphi_1(\xi,t)\sin\theta_0(\xi), \qquad \hat{T} = \frac{1}{\omega_1^2}\frac{\partial^2}{\partial t^2} + \frac{\omega_r}{\omega_1^2}\frac{\partial}{\partial t},$$
$$\sigma = (\beta_1 - \tilde{\beta}_2)/\tilde{\beta}_2,$$

$\omega_1 = c/y_0 = gM_0\sqrt{\tilde{\beta}_2\delta}/2$ is the activation frequency of the lower spin-wave mode and $\omega_r = \lambda\delta g M_0/4$ is the characteristic relaxation frequency.

The operator $\hat{L}$ has the form of a Schrödinger operator with a non-reflecting potential:

$$\hat{L} = -y_0^2 \frac{{\rm d}^2}{{\rm d}\xi^2} + 1 - \frac{2}{ch^2(\xi/y_0)}.$$

The spectrum and the eigenfunctions of $\hat{L}$ are well known. It has one discrete level with eigenvalue $\lambda_0 = 0$ corresponding to a localized wavefunction

$$f_0(\xi) = \frac{1}{\sqrt{2y_0}\,ch(\xi/y_0)}$$

and also a continuous spectrum $\lambda_p = 1 + p^2 y_0^2$ corresponding to the eigenfunctions

$$f_p(\xi) = \frac{1}{b_p\sqrt{L}}\left(\tanh\frac{\xi}{y_0} - {\rm i}py_0\right)\exp({\rm i}p\xi),$$

where $b_p = \sqrt{1 + p^2 y_0^2}$ and L is the crystal length.

We seek the solution of the system of equations of the first approximation (10) and (11) as an expansion over a complete orthonormalized set of the eigenfunctions $\{f_0(\xi), f_k(\xi)\}$:

$$\theta_1(\xi,t) = {\rm Re}\left\{\sum_p [c_p^{(1)} f_p(\xi) + c_0^{(1)} f_0(\xi)]\exp[{\rm i}(ky - \omega t)]\right\},$$
$$\varphi_1(\xi,t) = {\rm Re}\left\{\sum_p [d_p^{(1)} f_p(\xi) + d_0^{(1)} f_0(\xi)]\exp[{\rm i}(ky - \omega t)]\right\}.$$

For a monochromatic external magnetic field of frequency ω, with all three components different from zero, we obtain from equations (10) and (11)

$$\begin{aligned}\theta_1(\xi,t) &= a_1(t)G_1(\xi) + a_2(t)G_2(\xi),\\ \mu_1(\xi,t) &= a_3(t)\cos\theta_0(\xi) + a_4(t)\sin\theta_0(\xi)\\ &\quad + a_5(t)G_3(\xi).\end{aligned} \tag{12}$$

Here we introduce the following notations:

$$a_1(t) = -\frac{R\rho b_2}{4\tilde{\beta}_2}E_y, \qquad a_2(t) = -\frac{2}{\tilde{\beta}_2 g M_0 \delta}\dot{h}_z,$$

$$a_3(t) = \frac{\rho\pi d g M_0 h_x + 4\dot{h}_x}{\tilde{\beta}_2 g M_0 \delta[\sigma - q_1 + {\rm i}q_2]},$$

$$a_4(t) = \frac{-4\dot{h}_y}{\tilde{\beta}_2 g M_0 \delta[1 + \sigma - q_1 + {\rm i}q_2]}, \qquad a_5(t) = \frac{dh_x}{\tilde{\beta}_2\delta},$$

$$G_1(\xi) = y_0\int_{-\infty}^{+\infty} \frac{\cos(p\xi)\tanh(\xi/y_0) + (py_0)\sin(p\xi)}{ch(\pi p y_0/2)} \times \frac{{\rm d}p}{\Omega_1(p,\omega)},$$

$$G_2(\xi) = y_0\int_{-\infty}^{+\infty} \frac{\sin(p\xi)\tanh(\xi/y_0) - (py_0)\cos(p\xi)}{sh(\pi p y_0/2)} \times \frac{{\rm d}p}{\lambda_p\Omega_1(p,\omega)},$$

$$G_3(\xi) = y_0\int_{-\infty}^{+\infty} \frac{\sin(p\xi)\tanh(\xi/y_0) - (py_0)\cos(p\xi)}{sh(\pi p y_0/2)} \times \frac{{\rm d}p}{\Omega_2(p,\omega)},$$

where

$$q_1 = (\omega/\omega_1)^2, \qquad q_2 = (\omega\omega_r/\omega_1^2),$$

$$\Omega_1(p,\omega) = \lambda_p - q_1 + {\rm i}q_2,$$

$$\Omega_2(p,\omega) = \lambda_p(\lambda_p + \sigma - q_1 + {\rm i}q_2).$$

On the basis of the requirement of the vanishing of the Goldstone mode amplitude ($d_0^{(1)} = 0$) [16], we come to the equation for the definition of DW velocity:

$$\dot{V}_1 + \omega_r V_1 = -\frac{Ry_0 g M_0}{2}(\pi\rho\dot{h}_z + dgM_0 h_z). \tag{13}$$

The solution of this equation describes the DW oscillations in an external oscillating field and has the form

$$Y(t) = \mathrm{Re}\left[\frac{Ry_0gM_0}{2}\frac{\mathrm{i}dgM_0 - \pi\rho\omega}{\omega(\omega_r + \mathrm{i}\omega)}h_{0z}\exp[\mathrm{i}(\omega t + \chi_z)]\right]. \quad (14)$$

Let us separate the real part in the expression (14). Then we can rewrite the solution in the following form:

$$Y(t) = A\cos(\omega t + \chi_0), \quad (15)$$

where $A = \frac{y_0gM_0h_{0z}}{2\omega}\sqrt{\frac{(dgM_0)^2+(\pi\omega)^2}{\omega^2+\omega_r^2}}$ is the DW oscillation amplitude and χ_0 is the initial phase shift.

The DW drift motion is a second-order effect relative to the field amplitude. Consequently, the DW drift velocity is defined from the equation of the second order of perturbation theory:

$$\begin{aligned}(\hat{L} + \hat{T})\theta_2(\xi, t) &= -\frac{R\cos\theta_0}{y_0\omega_1^2}(\dot{V}_2 + \omega_r V_2) \\ &+ \frac{\theta_1'}{\omega_1^2}(\dot{V}_1 + \omega_r V_1) + \frac{2V_1}{\omega_1^2}\dot{\theta}_1' \\ &+ \frac{2d}{\tilde{\beta}_2\delta}(\varphi_1 h_x\cos\theta_0 + \theta_1 h_z\sin\theta_0) \\ &+ \frac{\cos 2\theta_0}{\tilde{\beta}_2}\left(b_2E_y\theta_1 - \frac{4}{\delta}h_xh_y\right) \\ &+ \frac{4}{\tilde{\beta}_2\delta gM_0}(\varphi_1\dot{h}_x + 2\dot{\varphi}_1h_x\sin^2\theta_0) \\ &+ \frac{\sin 2\theta_0}{2}\left[\frac{V_1^2}{c^2} - \frac{4}{\tilde{\beta}_2\delta}(h_x^2 - h_y^2) - (\sigma+1)\varphi_1^2 + \frac{(\dot{\varphi}_1)^2}{\omega_1^2}\right. \\ &\left. - y_0^2(\varphi_1')^2 - 2\theta_1^2 + \frac{8}{\tilde{\beta}_2\delta gM_0}h_y\dot{\varphi}_1\right], \end{aligned} \quad (16)$$

where a prime denotes the differentiation with respect to variable ξ.

The second equation of the system, which follows from equation (6) and defines the function $\varphi_2(\xi, t)$, has a similar structure, but does not contain a second-order term in the expansion of the DW velocity (V_2) and will therefore be of no interest. Since we are interested only in forced motion of DW, then for the determination of the velocity $V_2(t)$ it is enough to find the coefficient $d_0^{(2)}$, corresponding to the Goldstone mode in the expansion of $\theta_2(\xi, t)$ by its own eigenfunctions of the operator $\hat{L}$, and to equate it to zero. Substituting the functions $\theta_1(\xi, t)$ and $\varphi_1(\xi, t)$ (12) into equation (16), averaging it over the oscillation period and integrating, we come to the following expression for the drift velocity $V_{\mathrm{dr}} = \bar{V}_2$:

$$\begin{aligned}V_{\mathrm{dr}} &= \nu_0R[\rho A_1(\omega;\chi) + D_1(\omega;\chi)]H_{0x}H_{0y} \\ &+ \tilde{\nu}_0R[\rho A_2(\omega;\chi_z) + D_2(\omega;\chi_z)]H_{0z}E_{0y}. \end{aligned} \quad (17)$$

Here we introduce the notations:

$$A_1(\omega;\chi) = \frac{\pi}{4}\frac{q_1q_2}{Q_1}[q_2\cos\chi - (B_1B_2 + q_2^2)\sin\chi],$$

$$\begin{aligned}D_1(\omega;\chi) &= \sqrt{\frac{d^2}{\tilde{\beta}_2\delta}}\frac{\sqrt{q_1}}{2Q_1}[q_2(\eta_1(B_2^2 + q_2^2) + \eta_2B_2)\cos\chi \\ &- (B_1B_2^2 + q_2^2(B_1 - \eta_2))\sin\chi],\end{aligned}$$

$$\begin{aligned}A_2(\omega;\chi_z) &= \frac{\omega}{4gQ_2}[q_2(\omega_1^2\eta_2 + \omega^2\eta_3)\cos\chi_z \\ &- (\omega_r^2\eta_4 + \omega^2\eta_3(1 + q_1))\sin\chi_z],\end{aligned}$$

$$D_2(\omega;\chi_z) = \frac{\pi}{16}\frac{dM_0\omega_r}{Q_2}[\omega_r\cos\chi_z + \omega(1 - q_1)\sin\chi_z],$$

$$Q_1 = [B_1B_2 + q_2^2]^2 + q_2^2, \quad B_1 = 1 + \sigma - q_1, B_2 = \sigma - q_1,$$

$$Q_2 = [(1 - q_1)^2 + q_2^2](\omega^2 + \omega_r^2),$$

$\tilde{\nu}_0 = \frac{b_2}{\tilde{\beta}_2}\nu_0$, $\nu_0 = \frac{g^2y_0}{\omega_r}$ are the motilities of the DW; $\chi = \chi_x - \chi_y$ is the comparative phase displacement; $\eta_1 \approx -0.5$, $\eta_2 \approx 2.5$, $\eta_3 \approx 0.1$, $\eta_4 = 2.6$ are the numerical parameters.

It should be noted that $A_1(\omega;\chi)$ and $D_1(\omega;\chi)$ are dimensionless quantities, and $A_2(\omega;\chi_z)$ and $D_2(\omega;\chi_z)$ have the units Oe.

4. Discussions

Firstly, let us discuss certain features of solutions (12) and (14) of the first-order equations (10) and (11).

The eigenfunctions of the operator $\hat{L}$ were obtained by Winter [17] in the problem on spin excitations of magnetics. In 180° DW spins can participate in oscillations of two types. The first type of oscillation is related directly with DW. These oscillations are referred to as the intra-wall oscillations and correspond to the localized wavefunction $f_0(\xi)$. The second type of oscillation is the analogue of common spin waves inside domains. These oscillations correspond to the continuous spectrum which is described by the wavefunctions $f_p(\xi)$.

It follows from the relations (12) and (14) that all components of an external magnetic field and the electric field component E_y excite the second type of oscillations, while the component h_y excites only the state with $p = 0$. The components h_x and h_z also excite the oscillations of the first type. There is another situation in DW of **ac**-type: all the components of a magnetic field participate in the intra-wall oscillations; the field components h_x, h_z and E_y excite the state of the continuous spectrum with $p = 0$, and the component h_y excites all the intra-domain oscillations.

The features of oscillatory motion of DW are the consequence of the fact that the electric field in the linear approximation does not cause the motion of the **ab**-type DW. The DW of **ac**-type [10] behaves similarly, while a variable electric field (as was mentioned above) excites oscillations of 90° DW near the spin-reorientation phase transition [9].

From the relation (15) it is easy to find the velocity of the oscillatory motion of DW: $V = \omega A$. The obtained result is accurate for the mobility of DW for an oscillatory regime of motion in a magnetic field [18]. The comparison of the amplitude A of DW oscillations with the experimental data results in a good agreement: as for the frequencies of the experiment, we have $dgM_0 \gg \omega_r \gg \omega$; then the dependence of the amplitude on frequency is equal to $A \sim 1/\omega$, which is observed experimentally [19].

Let us discuss now the features of drift motion of DW. The analysis of equation (17) shows that the DW drift velocity is defined by terms of two types. The terms of the first type $D_1(\omega;\chi)$ and $D_2(\omega;\chi_z)$ are due to a Dzyaloshinskii

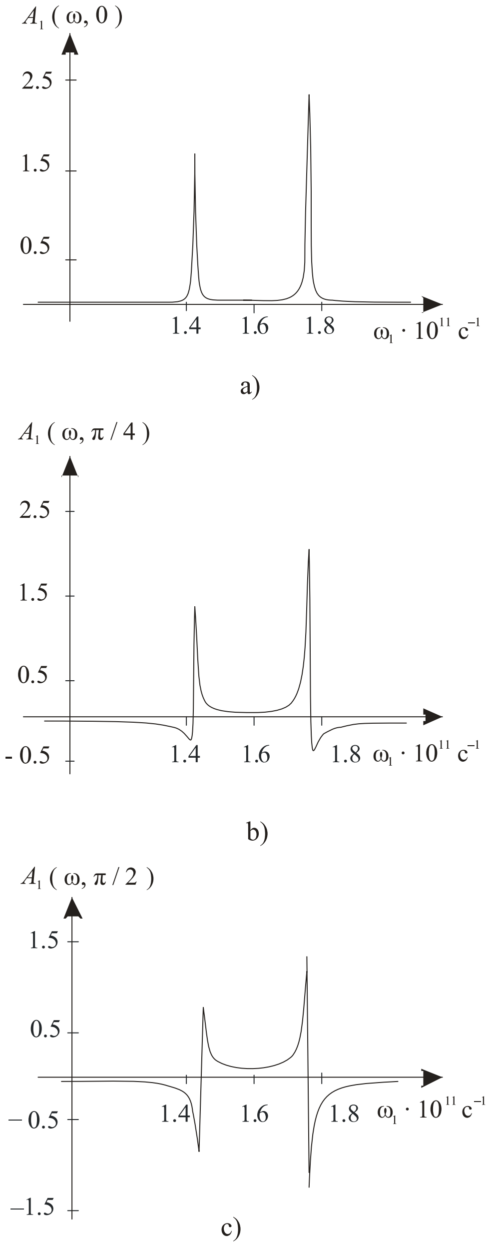

Figure 1. Dependences of $A_1(\omega;\chi)$ on frequency of the external field at $\chi = 0$ (a), $\chi = \pi/4$ (b) and $\chi = \pi/2$ (c).

interaction, and the second type terms $A_1(\omega;\chi)$ and $A_2(\omega;\chi_z)$ are present also in a pure antiferromagnet. For an estimation of the contributions of these terms to the drift velocity of DW for different values of the frequency and phase shifts, we will use the characteristic values of the parameters of Seignette magnetics [13]: $d \sim 10^2$, $\sigma = 2$, $y_0 = 5\times10^{-6}$ cm, $M_0 = 200$ Oe, $g = 1.76\times10^7$ s^{-1} Oe^{-1}, $\omega_r \sim 10^9$ s^{-1}, $\omega_1 \sim 10^{11}$ s^{-1}, and $\frac{b_2}{\beta_2} \sim 10^{-4}$. Then the mobility of DW is equal to $\nu_0 \approx 1.55$ cm s^{-1} ($\tilde{\nu}_0$, accordingly, is four orders less).

Let us analyse the dynamics of DW in the example of the term $A_1(\omega;\chi)$ which determines the most typical features of DW drift velocity. The dependence $A_1(\omega;\chi)$ on the frequency of an external magnetic field is presented in figure 1 for different values of phase shifts $\{\chi = 0, \chi = \pi/4, \chi = \pi/2\}$ in the field $H_{0x} = H_{0y} = 1$ Oe.

Table 1. Dependence of the drift velocity of DW (due to the term $D_1(\omega;\chi)$) on resonance frequencies and polarization at $H_{0x} = H_{0y} = 1$ Oe.

	ω	
$V_{dr}(\omega;\chi)$ cm s^{-1}	$\omega_1\sqrt{\sigma}$	$\omega_1\sqrt{\sigma+1}$
$V_{dr}(\omega;0)$	±1.4	-232.0
$V_{dr}(\omega;\frac{\pi}{2})$	1.6	∓40.4
$V_{dr}(\omega;\frac{\pi}{4})$	1.5	-166.8 4.9

Two typical resonances on the frequencies $\omega = \omega_1\sqrt{\sigma}$ and $\omega = \omega_1\sqrt{\sigma+1}$ take place in the case $\chi = 0$. Thus $A_1(\omega_1\sqrt{\sigma};0) \approx 1.6$ and $A_1(\omega_1\sqrt{\sigma+1};0) \approx 2.4$ which provides the absolute values 2.4 and 3.7 cm s^{-1} of DW drift velocity, accordingly.

In the case $\chi = \pi/4$ the resonance–antiresonance behaviour of the function $A_1(\omega;\pi/4)$ holds, but becomes asymmetrical. The resonances in those regions of the dependence which took place at $\chi = 0$ (the area of positive values of the function A_1) remain pronounced. The width of the resonance–antiresonance region in this case is equal to $\Delta\omega \approx 1, 4\times10^9$. The function $A_1(\omega;\frac{\pi}{4})$ possesses the values

$$A_1\left(\omega_1\sqrt{\sigma};\frac{\pi}{4}\right) \approx \begin{cases} -0,2 \\ 1,3 \end{cases},$$

$$A_1\left(\omega_1\sqrt{\sigma+1};\frac{\pi}{4}\right) \approx \begin{cases} 2 \\ -0,4. \end{cases}$$

The maximum value of the drift velocity (3.1 cm s^{-1}) in this case is achieved at the frequency $\omega_1\sqrt{\sigma+1}$.

In the case $\chi = \pi/2$ the peculiarities of the type 'resonance–antiresonance' arise at the same frequencies, and $A_1(\omega_1\sqrt{\sigma};\pi/2) \approx \mp0.8$ and $A_1(\omega_1\sqrt{\sigma+1};\pi/2) \approx \pm1.2$. The absolute values of drift velocity 1.2 and 1.8 cm s^{-1} correspond to these values, accordingly. Near these frequencies the DW changes the direction of motion into the opposite one. The transition between the resonance and antiresonance behaviours occurs in a narrow region of frequencies which is of the same order for both peculiarities and is equal to $\Delta\omega \approx 10^9$ s^{-1}.

The obtained results of the contribution of the term $A_1(\omega;\chi)$ into the drift velocity were obtained at $H_{0i} = 1$ Oe. However the criterion of smallness of the amplitude in the perturbation theory holds and for $H_{0i} = 10$ Oe: $h_{0i} = H_{0i}/M_0 = 0.05 \ll 1$. In the field $H_{0i} = 10$ Oe the values of DW drift velocity turn out to be two orders higher.

The results of the analysis of other terms, $D_1(\omega;\chi)$, $A_2(\omega;\chi_z)$ and $D_2(\omega;\chi_z)$, are presented in tables 1 and 2. For those cells in the table, in which one number is indicated, there is a simple resonance of the corresponding frequency. If two numbers are indicated, the resonance–antiresonance character of the dependence is observed. The signs '±' before the values denote that the dependence is symmetric and the resonance starts from the positive value of the velocity, and then the antiresonance (the negative value) follows. Two different values in a cell indicate the asymmetrical character of the resonance–antiresonance dependence.

Table 2. Dependence of the drift velocity of DW (due to the terms $A_2(\omega;\chi_z)$ and $D_2(\omega;\chi_z)$) on resonance frequency and polarization at $H_{0z} = 10$ Oe, $E_{0y} = 0.1$ CGSE units.

$V_{dr}(\omega;\chi)$ cm s^{-1}	$A_2(\omega_1;\chi_z)$	$D_2(\omega_1;\chi_z)$
$V_{dr}(\omega_1;0)$	56.5	0.6
$V_{dr}(\omega_1;\frac{\pi}{2})$	−438.9	±0.3
$V_{dr}(\omega_1;\frac{\pi}{4})$	−270.4	0.5 −0.1

In the limiting case of small frequencies ($\omega \ll \omega_r$) the drift velocity of DW for $H_{0z} = 10$ Oe and $E_{0y} = 0.1$ CGSE units is equal to

$$V_{dr} = \tilde{v}_0 R\left[\frac{\pi}{16} d M_0 \cos\chi_z\right] H_{0z} E_{0y}\bigg|_{\chi_z=0} \approx 0.6 \text{ cm s}^{-1}.$$

In this frequency range the contributions of other field components are negligible.

It follows from the presented results that, in the presence of a purely magnetic field in the XY plane, the maximum effect of drift is caused by Dzyaloshinskii interaction. The term $D_1(\omega;\chi)$ takes its maximum value on the resonant frequency $\omega_1\sqrt{\sigma+1}$ for $\chi = 0$. The maximum contribution to the drift velocity due to $H_{0z}E_{0y}$ is provided by the term $A_2(\omega;\chi_z)$. At the resonance frequency $\omega = \omega_1$ for the values $H_{0z} = 10$ Oe, $E_{0y} = 0.1$ CGSE units and $\chi = \pi/2$, the drift velocity can reach the value of 4 m c^{-1}.

The character of the dependence of drift velocity (17) on DW topological charge R and parameter ρ indicates the possibility of the drift of a stripe-domain structure (DS) formed by 180° DW. As the topological charges R in adjacent DW are different, the drift of DS caused by the terms $D_{1,2}(\omega,\chi)$ in a weak ferromagnet is impossible. Nevertheless it is possible in a pure antiferromagnet. For the drift of a stripe DS it is also necessary that the parameters ρ in adjacent DW are to be different, i.e. the orientations of the vector $\boldsymbol{l}$ in adjacent DW are to be opposite, but to have the same directions of rotation. In this case the factor $R\rho$ for adjacent DW has the same signs, and the DW move in one and the same direction, i.e. the motion of DS takes place.

The similar effect, namely the drift of a stripe DS in crossed electric and magnetic alternating fields, was predicted in [10].

5. Conclusions

We investigated the nonlinear dynamics of DW of **ab**-type in magnetic materials with linear magnetoelectric interaction in external alternating fields. It is established that, against the background of DW fast oscillations, a slow component of translatory (drift) motion of DW arises. The drift motion of DW can be caused either by the crossed alternating magnetic field polarized in the XY plane or by the crossed electric E_{0y} and magnetic H_{0z} fields. The drift velocity is formed by terms of two types: the terms of the first type are due to Dzyaloshinskii interaction, and the other type terms have an antiferromagnetic origin. The first type terms provide a maximum contribution for the drift in a magnetic field and the second type terms in crossed electric and magnetic fields.

The possibility of the drift of DS is predicted. The drift of a stripe DS is possible in a magnet with pure antiferromagnetic character of ordering under certain coordination of signs of the topological charge R and parameter ρ describing the turn of the vector of antiferromagnetism in DW.

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